

The Liouville Theorem and linear operators satisfying the maximum principle

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The classical Liouville Theorem states that bounded harmonic functions are constant. The talk will revisit this result for the most general class of linear operators with constant coefficients satisfying the maximum principle (characterized by Courrège in [2]). The class includes local and nonlocal and not necessarily symmetric operators among which you can find the fractional Laplacian, Relativistic Schrödinger operators, convolution operators, CGMY, as well as discretizations of them.

We give a full characterization of the operators in this class satisfying the Liouville property. When the Liouville property does not hold, we also establish precise periodicity sets of the solutions.

The techniques and proofs of [1] combine arguments from PDEs, group theory, number theory and numerical analysis (and still, they are simple, short, and very intuitive).

References

- [1] N. Alibaud, F. del Teso, J. Endal, and E. R. Jakobsen. *The Liouville theorem and linear operators satisfying the maximum principle*. Journal de Mathématiques Pures et Appliquées, 142:229–242, 2020.
- [2] P. Courrège. *Générateur infinitésimal d'un semi-groupe de convolution sur R^n , et formule de Lévy-Khinchine*. Bull. Sci. Math. (2), 88:3–30, 1964.

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