This document provides the mathematical model for the hybrid production-remanufacturing system (HPRS) of domestic electric storage waters heaters (DESWH), which is introduced in:

Devoto C, Fernández E, 2019, "*Recovery of used products and an application in the Uruguayan industry*" (in spanish), Bachelor thesis, Faculty of Engineering, Universidad de la República, Uruguay, https://hdl.handle.net/20.500.12008/20592.

The model considers the costs for holding inventories and for carrying out the production activities related to the new products and the returns of diverse qualities (inspection, remanufacturing and discard). Raw materials are not considered.

Sets:

- TC: Set of DESWH with copper storage tank.
- TA: Set of DESWH with steel storage tank.
- $T \ = TC \cup TA$: Set of all DESWH.
- TP: Set of DESWH that can be produced in the same period.
- MP: Set of raw materials.
- $MPD \subset MP$: Set of raw material with lot constraints.
- PR: Set of suppliers.
- PRM: Set of pairs (p, m) with $p \in PR$ and $m \in MP$ if material m is provided by supplier p.
- I: Set of periods.
- QR: set of remanufacturable qualities.
- QD: set of discard (non-remanufacturable) qualities.
- $Q = QR \cup QD$: set of all nominal qualities for grading incoming returns.
- $QB \subset Q$: a subset of qualities.
- $QA = Q \setminus QB$.

Parameters:

- C_t^p: Unit cost of production for product $t \in T$.
- C_{qt}^r: Unit cost of remanufacturing for product $t \in T$ from a return of quality $q \in QR$.
- C_{qt}^d: Unit cost of discarding for product $t \in T$ from a return of quality $q \in QD$.
- G_{qt}^d: Rescue value of discarding a product $t \in T$ from a return of quality $q \in QD$.
- K_t^p: Set-up cost of production for product $t \in T$.
- K_{qt}^r: Set-up cost of remanufacturing for product $t \in T$ from a return of quality $q \in QR$.
- K_{qt}^d: Set-up cost of discarding for product $t \in T$ from a return of quality $q \in QD$.
- K^e: Set-up cost of inspection the incoming returns.
- C_t^e: Unit cost of inspection for a return of product $t \in T$.
- H^u: Unit cost for holding inventory of uninspected returns.
- H_q^e: Unit cost for holding inventory of inspected-and-graded returns of quality $q \in Q$.
- H^s: Unit cost of inspection for final products.

- D_{ti}: Demand of product $t \in T$ in period $i \in I$.
- U_{ti}: Returns of product $t \in T$ in period $i \in I$.
- α_{qt}: Proportion of returns of product $t \in T$ with quality $q \in Q$.
- TN_t^p: Production time for product $t \in T$.
- TN_{qt}^r: Remanufacturing time for product $t \in T$ from a return of quality $q \in QR$.
- TN_{qt}^d: Discarding time for product $t \in T$ from a return of quality $q \in QD$
- TD^p: Total available time for production.
- TD^u: Total available time for recovery.
- QP: Storage capacity for final products.
- Io_t: Initial inventory level of product $t \in T$.
- A: A big number, with $A = \sum_{t\in T}\sum_{i\in I} D_{ti}$.
- θ: Real value in the range [0,1].
- B: Value for rounding (e.g. 0.5).

Variables:

- x_{ti}^p: Production quantity of product $t \in T$ in period $i \in I$.
- x_{qti}^r: Reman. quantity of product $t \in T$ from a return of quality $q \in QR$ un period $i \in I$.
- x_{qti}^d: Discarding quantity of product $t \in T$ from a return of quality $q \in QD$ in period $i \in I$.
- s_{ti}^s: Inventory level of a serviceable product $t \in T$ in period $i \in I$.
- s_{ti}^u: Inventory level of a returns of product $t \in T$ of quality $q \in Q$ in period $i \in I$.
- s_{qti}^e: Inventory level of an inspected return of product $t \in T$, quality $q \in Q$ in period $i \in I$.
- w_{ti}: Fractional quantity of returns of product $t \in T$ inspected in period $i \in I$.
- w_{qti}^{int}: Number of inspected returns of product $t \in T$ with quality $q \in Q$ in period $i \in I$.
- δ_{ti}^p: 1 if product $t \in T$ is produced in period $i \in I$, 0 otherwise.
- δ_{qti}^r: 1 if a return of product $t \in T$ and quality $q \in QR$ is reman. in period $i \in I$, 0 otherwise.
- δ_{qti}^d: 1 if a return of product $t \in T$ and quality $q \in QD$ is discarded in $i \in I$, 0 otherwise.
- ε_i: 1 if inspection of returns occurs in period $i \in I$, 0 otherwise.

Mixed-Integer Linear Programming formulation for the HPRS:

$$\text{Min} \quad \sum_{i\in I}\Big\{\sum_{t\in T}\Big(C_t^p x_{ti}^p + K_t^p \delta_{ti}^p + \sum_{q\in QR}(C_{qt}^r x_{qti}^r + K_{qt}^r \delta_{qti}^r) + \sum_{q\in QD}((C_{qt}^d - G_{qt}^d)x_{qti}^d + K_{qt}^d \delta_{qti}^d) + H^u s_{ti}^u + \sum_{q\in Q}(H_q^e s_{qti}^e) + H^s s_{ti}^s + C_t^e w_{ti}\Big) + K^e \varepsilon_i\Big\} \quad (1)$$

subject to:

$$x_{ti}^p \le A\delta_{ti}^p, \quad \forall t \in T,\ \forall i \in I \quad (2)$$

$$x_{qti}^r \le A\delta_{qti}^r, \quad \forall q \in QR, \forall t \in T,\ \forall i \in I \quad (3)$$

$$x_{qti}^d \le A\delta_{qti}^d, \quad \forall q \in QD, \forall t \in T,\ \forall i \in I \quad (4)$$

$$w_{ti} \le A\varepsilon_i, \quad \forall t \in T,\ \forall i \in I \tag{5}$$

$$\sum_{t\in T} x_{ti}^{p}\, TN_t^{p} \le TD^{p}, \quad \forall i \in I \tag{6}$$

$$\sum_{t\in T} \left(\sum_{q\in QR} x_{qti}^{r}\, TN_{qt}^{r} + \sum_{q\in QD} x_{qti}^{d}\, TN_{qt}^{d}\right) \le TD^{u}, \quad \forall i \in I \tag{7}$$

$$s_{ti}^{u} = s_{(t,i-1)}^{u} + U_{ti} - w_{ti}, \quad \forall t \in T,\ \forall i \in I \tag{8}$$

$$s_{qti}^{e} = s_{(q,t,i-1)}^{e} + w_{qti}^{int} - x_{qti}^{r}, \quad \forall q \in QR, \forall t \in T,\ \forall i \in I \tag{9}$$

$$s_{qti}^{e} = s_{(q,t,i-1)}^{e} + w_{qti}^{int} - x_{qti}^{d}, \quad \forall q \in QD, \forall t \in T,\ \forall i \in I \tag{10}$$

$$s_{ti}^{s} = s_{(t,i-1)}^{s} - D_{ti} + x_{ti}^{p} + \sum_{q\in QR} x_{qti}^{r}, \quad \forall t \in T,\ \forall i \in I \tag{11}$$

$$s_{t,0}^{u} = 0, \quad \forall t \in T \tag{12}$$

$$s_{q,t,0}^{e} = 0, \quad \forall q \in Q, \forall t \in T \tag{13}$$

$$s_{t,0}^{s} = Io_t, \quad \forall t \in T \tag{14}$$

$$\sum_{t\in T} s_{t,i}^{s} \le QP, \quad \forall i \in I \tag{15}$$

$$\sum_{tc\in TC} \delta_{tc,i}^{p} \le 1, \quad \forall i \in I \tag{16}$$

$$\sum_{ta\in TA} \delta_{ta,i}^{p} \le 1, \quad \forall i \in I \tag{17}$$

$$\sum_{tx\in TA\setminus\{ta\}} \sum_{tp\in TP[tx]} \delta_{tp,i}^{p} \le 1 - \delta_{ta,i}^{p}, \quad \forall ta \in TA, \forall i \in I \tag{18}$$

$$w_{qti}^{int} \le \alpha_{qt} w_{ti} + B, \quad \forall q \in Q, \forall t \in T,\ \forall i \in I \tag{19}$$

$$w_{qti}^{int} \ge \alpha_{qt} w_{ti} + B - \theta, \quad \forall q \in Q, \forall t \in T,\ \forall i \in I \tag{20}$$

$$w_{qti}^{int} = w_{ti} - \sum_{q\in QA} w_{qti}^{int}, \quad \forall q \in QB, \forall t \in T,\ \forall i \in I \tag{21}$$

$$s_{ti}^{s}, s_{ti}^{u}, s_{qti,}^{e}, x_{ti}^{p} \ge 0,\ w_{ti}, w_{qti}^{int} \in \aleph^{+} \cup \{0\},\ \delta_{ti}^{p}, \varepsilon_i \in \{0,1\}, \quad \forall q \in Q, t \in T, \forall i \in I \tag{22}$$

$$x_{qti}^{r} \ge 0,\ \delta_{qti}^{r} \in \{0,1\}, \quad \forall q \in QR, t \in T, \forall i \in I \tag{23}$$

$$x_{qti}^{d} \ge 0,\ \delta_{qti}^{d} \in \{0,1\}, \quad \forall q \in QD, t \in T, \forall i \in I \tag{24}$$

The objective function of (1) is for minimizing the sum of the costs involved of production, inspection, remanufacturing, discard and holding inventories. Constraints (2) to (5) are to activate the binary variables related to the production activities. Constraints (6) and (7) state the recovery capacity in terms of time. Constraints (8) to (11) are the well-known inventory balance equations for incoming returns, remanufacturable and non-remanufacturable returns, and final products, respectively. Constraints (12) to (14) state that all initial inventory levels are equal to zero. Constraints (15) state the storage capacity for final products. We note that there are not capacity restrictions for returns, as it is assumed that the recovery options are not yet implemented in practice. Constraints (16) to (18) are for establishing the types of DESWH that can be produced in the same period. Constraints (19) to (21) are for obtaining the number of inspected returns according to the factor qualities. Constraints (22) to (24) state the set of values for the decision variables.