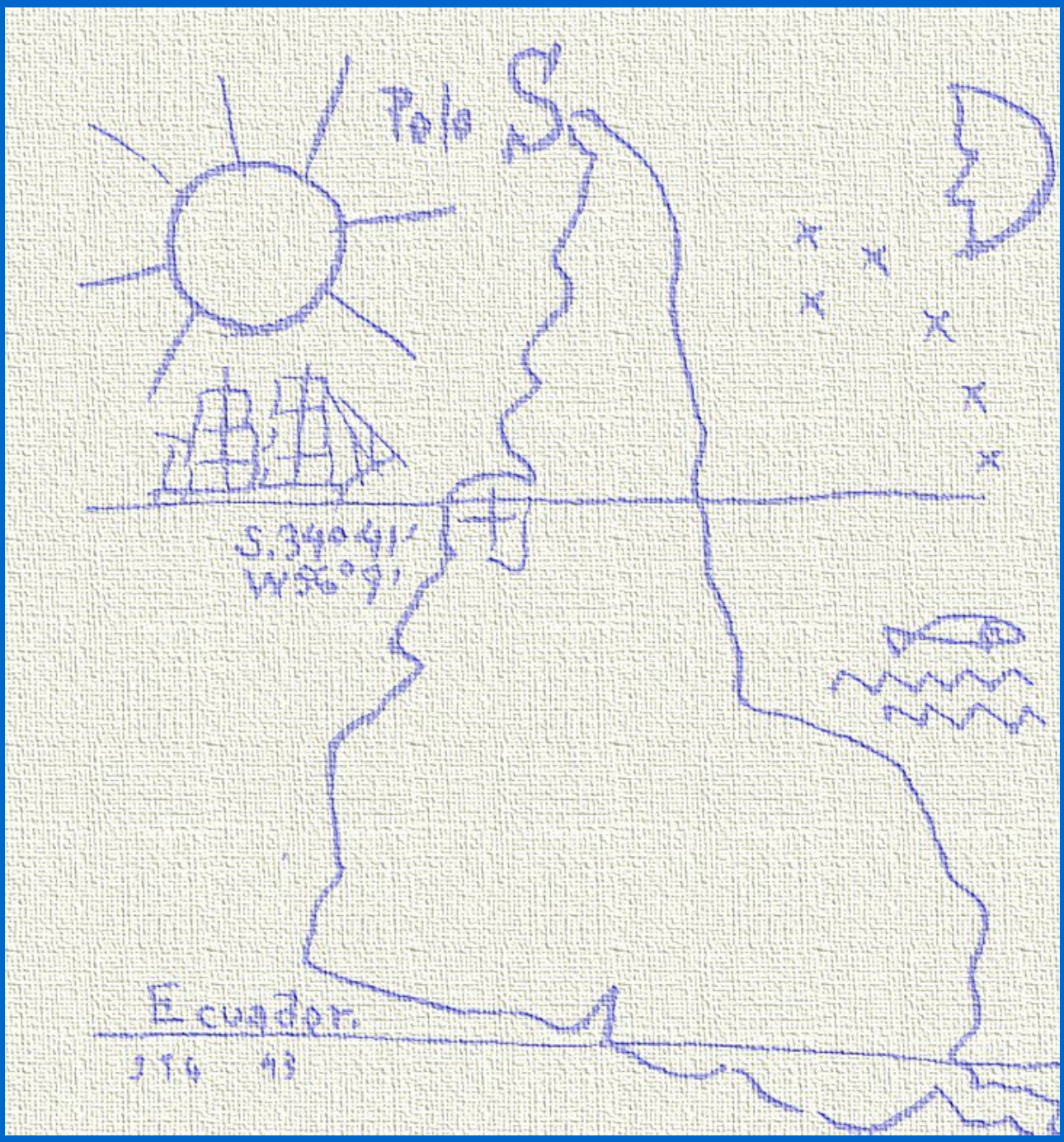


# BOOK OF ABSTRACTS

## MONTEVIDEO DYNAMICAL SYSTEMS CONFERENCE 2012

CSIC · Universidad de la República Uruguay · IMERL · Facultad de Ingeniería · ANII · ICTP · PRONEX Sistemas Dinâmicos IMPA Instituto do Milênio · antel · PEDECIBA · UTE · IFUM Instituto Franco-Uruguayo de Matemática

# MONTEVIDEO DYNAMICAL SYSTEMS CONFERENCE 2012

## from August 13th to 17th

## with a session in honour of Jorge Lewowicz*

### TOPICS

**Ergodic theory**

**Topological dynamics**

**Expansive dynamical systems**

**Lagrangian dynamics**

**N-body problem**

**Partially hyperbolic dynamics**

**Systems with singularities**

**Billiards**

**Group actions, foliations**

**Session of Dynamical Systems in Science and Technology**



***The Universidad de la República titles Dr. Jorge Lewowicz, "Doctor Honoris Causa" on August 15th, 2012**

“Book of Abstacts
Montevideo Dynamical Systems
Conference 2012”

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# Laudatio
# Honoris Causa Jorge Lewowicz

Martín Sambarino

Hoy es la ceremonia de entrega del título de Dr. Honoris Causa al Prof. Jorge Lewowicz y he sido designado para hacer la Laudatio; es un honor y un placer decir estas palabras en este homenaje a mi Profesor, colega y amigo Jorge.

Esta tarde, en el Congreso de Sistemas Dinámicos hemos escuchado dos conferencias, una por Rafael Potrie y otra por Rafael Ruggiero sobre los trabajos científicos de Jorge Lewowicz, su influencia y posterior desarrollo de su obra científica.

Me voy a referir a su legado científico, pero de forma mucho menos técnica, y tambien a otro aspecto de la obra del Prof. Lewowicz, que no es independiente de lo anterior: la continuación de la escuela matemática uruguaya originada por Massera y Laguardia y la conformación de un grupo numeroso y destacado de investigadores en Sistemas Dinámicos.

Jorge Lewowicz comenzó sus estudios de Ingeniería a mediados de la década del 50 e ingresa como ayudante del Instituto de Matemática y Estadística en el año 58. Bajo la orientación del Prof. Jose Luis Massera inició sus estudios sobre Ecuaciones Diferenciales y en 1961 publica su primer trabajo científico: Sobre un teorema de Szmydtowna.

Posteriormente, mediante una beca Fullbright, viaja a Estados Unidos en 1964 y en 1966 obtiene su título de PhD en Matemática en Brown University.

Desde sus inicios Lewowicz se interesa por cuestiones de estabilidad y de dinámica topológica, temas que ha desarrollado y plasmado en mas de una treintena de artículos científicos. Pero por sobre todas las cosas, es mayormente reconocido por su desarrollo de la teoría de sistemas expansivos, temas en los cuales Lewowicz hizo escuela. En el área de Sistemas Dinámicos, el Uruguay fue ampliamente reconocido por tener un grupo muy fuerte en dinámica de expansivos. Entre los reconocimientos académicos como científico, Lewowicz fue designado miembro de la Academia de Ciencias de Tercer Mundo, miembro correspondiente de la Academia de Ciencias de Argentina, y miembro de la Academia de Ciencias del Uruguay.

No voy a hacer aquí una lista ni ennumeración de sus trabajos. Sí voy a decir que

todos sus trabajos comparten las siguientes características: originalidad (y con originalidad me refiero además a una linea propia de invesitigación, a su busqueda íntima de la armonía de la Matemática), la interrelacion entre diversos métodos e ideas profundas en la matemática, y que además son de una gran elegancia y belleza.

De todas formas, voy a referirme a tres trabajos fundamentales de su obra en particular. Primero, a su artículo "Lyapunov functions and topological stability" publicado en Journall of Differential Equations en 1980. En este trabajo, Lewowicz introduce la noción de funciones de Lyapunov para sistemas dinámicos, y demuestra un teorema de estabilidad topológica bajo las condiciones de existencia de una función de Lyapunov no degenerada. Esto dió lugar a nuevos ejemplos, fuera del mundo hiperbólico, de sistemas topológicamente estables. Además, este resultado representa una analogía notable, e insospechada a priori, con el resultado de estabilidad asintótica de puntos de equilibrio para ecuaciones diferenciales. En el mismo trabajo también se caracterizan los difeomorfismos de Anosov o conjuntos hiperbólicos mediante una forma cuadrática no degenerada que crece a lo largo de las trayectorias. Esta caracterización y forma de pensar, ha sido usada y desarrollada despues por Roberto Markarian para el estudio de billares.

El segundo trabajo que me voy a referir es Persistence in Expanisve Systems, publicado en Ergodic Theory and Dynamical Systems en 1983. La expansividad es una propiedad que aparece naturalmente en difeomorfismos de Anosov y en conjuntos hiperbólicos y es el concepto básico de lo que hoy se conoce como caos o impredictibilidad. Su definición es extremadamente sencilla y general en su contexto. En este trabajo, Lewowicz comienza con el estudio sistemático de sistemas expansivos y prueba una propiedad esencial de estos: no hay puntos Lyapunov estables. También introduce la noción de persistencia, y prueba, bajo ciertas condiciones que no voy a explicar aquí, que los sistemas expansivos son persistentes.

Y en tercer lugar, quiero referirme a su artículo Expansive Homeormorphisms of Surfaces, publicado en 1989. Este es su principal contribución y es uno de los trabajos mas importantes y célebres del área, en particular en dinámica topológica. En este trabajo no solo resuelve un viejo problema abierto (sobre la no existencia de homeomorfismos expansivos en la esfera), sino que hace una clasificación completa de los homeomorfismos expansivos en superficies. En este trabajo Lewowicz exhibe de manera notable la dialéctica entre la topología y la dinámica. Pero mas allá de la importancia del resultado y su profundidad, tanto el enunciado, el resultado en sí, es de una gran belleza como lo es su demostración: a partir de una simple definición y a través de argumentos simples pero profundos se va tejiendo la relación entre la dinámica y la topología hasta llegar a una descripición de las propiedades de los sistemas expansivos que permiten su clasificación.... es como si fuera una sinfonía que comienza con un solo intrumento y este, a través de su melodía y armonía, va despertando y contagiando a toda la orquesta para el Grand Finale.

Como dije al principio, quiero referirme ahora a otro aspecto del legado académico de Lewowicz: su carácter como formador. Fiel a la tradiciones de la escuela matemática uruguaya, la formación y el estímulo a los jóvenes y a la iniciación de la investigación es y ha sido una de sus preocupaciones centrales. Fiel al rigor académico, estimuló con generosidad la salida de muchos estudiantes a realizar el doctorado en lugares de excelencia.

Ha dirigido 6 tesis de doctorado (2 en Brasil, 4 en Uruguay) y 15 tesis de maestría (8 en Venezuela, 7 en Uruguay) y una decena de monografía de Licenciatura. Pero mas allá de sus alumnos directos, Lewowicz ha influenciado el desarrollo como matemáticos e investigadores de otro tanto que no fueron sus doctorandos, tanto en el Uruguay como en el exilio. Hoy, nuestra Universidad cuenta con 17 investigadores en sistemas dinámicos y la conformación de este grupo se debe en gran o total medida a Jorge Lewowicz.

Lewowicz trasmitió a lo largo de estos años, tanto en sus clases como en las célebres caminatas por el pasillo del IMERL y en diversas reuniones, valores fundamentales de la escuela matemática uruguaya: la investigación temprana, la calidad y rigurosidad científica, el desarrollo de un ámbito propicio para la discusión, la trasmisión y creación de conocimiento, el compromiso social e institucional. Pero no solo dentro de la matemática, sino en la Facultad de Ingeniería y en la Universidad, estimuló, inspiró e influenció el quehacer científico de muchos jovenes y no tan jovenes, generación tras generación. Defendió y abogó por la calidad academica con estandares internacionales, tanto en la Facultad de Ingeniería como en nuestra Universidad, en particular desde la CSIC. La Facultad de Ingeniería es lo que es hoy en parte a la semilla del Instituto de Matematica y Estadística Prof. Rafael Laguardia, y de alguna forma el papel que jugaron Massera y Laguardia antes de la dictadura en la Facultad, lo desempeño Jorge despues de ésta.

Quiero contar una anécdota, que muy pocos conocen pero que ilustra las cosas que genera Jorge en su entorno. José Vieitez fue alumno de Jorge y el primer Doctor en Matemática por la Universidad de la República-PEDECIBA. Una tarde, llegando Jorge a su casa, abre el casillero de la correspondencia y se encuentra con el título de Dr de Jose Vieitez que había sido expedido ese día y con una dedicatoria en el reverso: Al Maestro con cariño.

Cuando uno entraba en una clase de Lewowicz inmediatamente se sorprendía. Se sorprendía por la pasion, amor y placer que tenía con la matemática. Se sorprendía por la profundidad y la trascendencia filosófica que le daba al objeto de estudio del curso. Y se sorprendía también por la importancia que le daba a los alumnos: parecía que no habia cosa mas importante en el mundo que las dudas e inquietudes que podrían tener estos y dejaba bien en claro que estaba a entera disposición en cualquier momento o lugar; hasta nos daba el número de telefono de su casa (que claro, muchas veces después no atendia!). Y se sorprendía también, si aún no lo conocía, por su singular personalidad, agudeza y sentido del humor.

Aquellos que tuvimos el privilegio de estar en una clase con Lewowicz, la disfrutamos minuto a minuto, incluso cuando luego de enunciar un resultado nos decía: “Señoras y señores, tienen 3 minutos para pensar la demostración”, se hacía un silencio total y Jorge caminaba de lado a lado del salón. Y si alguien esbozaba alguna idea para la demostración, entonces Jorge la seguía, sin importar si habia una camino mas corto o mas facil: lo mas importante era respetar la libertad y los caminos de pensamiento de cada uno.

Y como decía anteriormente, defendía la investigación temprana: uno no tenía que ser erudito para ser creativo, más aún, lo que había que estimular era la creatividad y en todo caso, la erudición venía de la mano de la necesidad de resolver y plasmar las ideas. Y así, les daba a los alumnos problemas abiertos o simplificaciones de estos, o incluso bajaba a tierra diversos problemas técnicos de sus investigaciones para que los alumnos los pudieran atacar. Y cuando alguien, fulano de tal, le hacía una pregunta de matemática que Lewowicz entendía que el mismo la podía responder, decía, no exento de picardía: esa es una pregunta que debe responder fulano de tal.

Hablar de matemática con Jorge ha sido y es iluminador y un placer, aún para los mas jóvenes, bien en el Instituto, bien en una reunion o bien en las visitas que recibe ahora en su casa. Y no solo de matemática, sino de lo que han sido sus preocupaciones durante toda su vida: la ciencia, la educación, la universidad, el pais, el ser humano. Y también, como hicieron otros matemáticos que volvieron del exilio, a través de anécdotas e historias, nos fue pincelando la figuras de Laguardia y Massera y de la vida del Instituto previo a la dictadura del 73, de forma que las generaciones mas jovenes aprendimos a respetar y querer entrañablemente, a sentirnos parte de una historia y a generar el compromiso de continuarla.

La Universidad debe reconocer, para si misma y para la sociedad, quienes son sus hombres de valía, y tu Jorge, vaya que sí lo sos.

Muchas gracias.

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# POSTER

## CONJUGATIONS OF ONE-DIMENSIONAL MONOTONE TRANSFORMATIONS WITH LIPSHITZ ONES

ANDREY ALPEEV

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Suppose G is at most countable semigroup of monotone continuous transformations of interval or circle. It will be shown, that this semigroup is topologically conjugated to the semigroup of Lipshitz transformations, i.e. there exists such a homeomorphism $f$, that for every $f \in G$ transformation $f^{-1}gf$ is Lipshitz. This is done by constructing a special quasi-invariant metric, inspired by Lyapunov metric. It means that every topological phenomenon, that can be observed on the countable semigroup of monotone continuous transformations of interval or circle, can be observed on the Lipshitz one too.

*Keywords:* Lipshitz map, topological conjugacy, one dimensional dynamics

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# STRUCTURE AND EXAMPLES OF PSEUDO-ANOSOV FLOWS IN GRAPHMANIFOLDS AND SEIFERT FIBERED PIECES.

THIERRY BARBOT
(JOINT WORK WITH SERGIO FENLEY)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

We describe an interaction of a pseudo-Anosov flow with possible Seifert fibered pieces in the torus decomposition of the underlying manifold: if the fiber is associated to a periodic orbit of the flow, we produce a standard form for the flow in the piece which is a neighborhood of finitely many weakly embedded Birkhoff annuli. A Birkhoff annulus is an annulus so that each boundary component is a closed orbit of the flow and the flow is transverse to the interior of the annulus. Using collections of Birkhoff annuli as a skeleton for some flows, we then produce a very large class of new examples of pseudo-Anosov flows in graph manifolds.

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# POSTER

## DISSIPATION AND SUPPRESSION OF THE FERMI ACCELERATION IN THE ELLIPTICAL OVAL-SHAPED BILLIARD WITH TIME-DEPENDENT BOUNDARY

RAFAEL AMATTE BIZÃO AND EDSON DENIS LEONEL

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

In 1949, Enrico Fermi proposed a model to describe the acceleration of cosmic rays. In his original paper, Fermi found out that charged particles could be accelerated indefinitely from interactions with oscillating magnetic fields in space. Since then, there are several models to study this phenomenon, which is now called as Fermi acceleration. In our work, we studied a time dependent model that corresponds to a mix between the elliptical and the oval-shaped billiard in order to understand the Fermi acceleration. We also studied the effect that a flight dissipation may cause on this billiard.

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# ROBUST VANISHING OF ALL CENTRAL LYAPUNOV EXPONENTS

JAIRO BOCHI

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

I will describe $C^2$-open sets of iterated function systems on arbitrary compact manifolds admitting fully supported ergodic measures all whose Lyapunov exponents vanish. I will also discuss consequences for partially hyperbolic maps. The proofs follow the strategy introduced by Gorodetski, Ilyashenko, Kleptsyn and Nalski, who considered one-dimensional center. In order to deal with higher dimensional center, it is convenient to study the dynamics in the flag bundle. This is a joint work with C. Bonatti and L.J. Diaz.

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# A PARTIALLY HYPERBOLIC GEODESIC FLOW WHICH IS NOT ANOSOV

FERNANDO ANTÔNIO DE ARAUJO CARNEIRO
(JOINT WORK WITH ENRIQUE PUJALS)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

In a joint work with Enrique Pujals [CP], we construct a partially hyperbolic geodesic flow which is not Anosov. It is based on a deformation of the metric of a locally symmetric space of nonconstant negative curvature.

Actually, we prove:

**Theorem 1.** *For some compact locally symmetric space $(M, g)$ whose sectional curvature takes values in the whole interval $[-1, -\frac{1}{4}]$, there is a metric $g^*$ in $M$ such that its geodesic flow is partially hyperbolic but not Anosov.*

The following corollary is given by the persistence of quasi-elliptic nondegenerate periodic orbits.

**Corollary 1.** *There is an open set $\mathcal{U}$ of metrics in the set of metrics of $M$ such that for $g \in \mathcal{U}$, the geodesic flow of $g$ is partially hyperbolic but not Anosov, for $(M, g)$ as in the previous theorem. There is also an open set $\mathcal{U}'$ of metrics such that for $g \in \mathcal{U}'$, the geodesic flow of $g$ is partially hyperbolic non-Anosov and with conjugate points.*

A classical Mañé theorem [M] says that if, for a geodesic flow of a Riemannian manifold there is an invariant Lagrangian subbundle, then this Riemannian manifold does not have conjugate points.

The existence of a partially hyperbolic non-Anosov geodesic flow with conjugate points implies that this theorem does not generalize to the case of invariant isotropic subbundles.

We also prove that if a metric of nonpositive curvature is not a Riemannian product and its geodesic flow is partially hyperbolic, then

its rank is one. Other obstructions to partial hyperbolicity of a geodesic flow are also analyzed.

Now I'm working on some other properties of this examples, as transitivity, robust transitivity and others dynamical properties. I also have some results on the relation of curvature and partial hyperbolicity of geodesic flows.

## References

[CP] Fernando Carneiro and Enrique Pujals, Partially hyperbolic geodesic flows, http://arxiv.org/abs/1110.1050.

[M] Ricardo Mañé, On a theorem of Klingenberg, Dynamical systems and bifurcation theory, Proc. Meet., Rio de Janeiro/Braz. 1985, Pitman Res. Notes Math. Ser. 160, 319-345 (1987).

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# A LOWER BOUND FOR TOPOLOGICAL ENTROPY AND SOME GENERIC PROPERTIES FOR SYMPLECTIC DIFFEOMORPHISMS

THIAGO CATALAN
(JOINT WORK WITH VANDERLEI HORITA)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We prove that a C1-generic symplectic diffeomorphism is either partially hyperbolic or the topological entropy is bounded from below by the supremum over the largest positive Lyapunov exponent of the periodic points. For partially hyperbolic diffeomorphisms we also give a lower bound for the topological entropy, but now by the supremum of the largest positive Lyapunov exponent of the periodic points restricted to the central direction. We also prove a C1-generic symplectic trichotomy by means of m-elliptic periodic points.

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# DIMENSIONALITY REDUCTION IN OPTIMAL OPERATION OF HYDROTHERMAL POWER GENERATION DYNAMICAL SYSTEMS (RESEARCH ADVANCES)

RUBEN CHAER

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

The optimal operation of dynamical systems of power generation involves an Operation Policy which is obtained by solving an optimization problem. The classic statement of this problem leads to a Stochastic Dynamical Programming - PDE with the known "Bellman's curse of dimensionality".

The system of electricity generation, considering the reservoirs of energy and the start and stop times of thermal power plants, is a dynamical system. Therefore, achieving an optimum operation of such a system, to supply the demand at the lowest possible future cost, suffers from the Bellman's curse of dimensionality. This curse forces to simplify the model of the system to be solved with the available computing power.

This communication presents the state of progress on the research of a technique for automatic dimensional reduction, and of its implementation on the simulation platform SimSEE.

**Keywords:** Dynamical system, optimal operation, stochastic simulation

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# POSTER

## ONE-PARAMETER FAMILIES OF BILLIARDS ON CONVEX CURVES (OVALS)

JOSUÉ GERALDO DAMASCENO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We describe some dynamical properties of one parameter families of billiards on convex curves (ovals) which are deformed by the curvature flow.

As the billiard table deforms, the ratio between minimal and maximal curvature converges to 1, and by a classical result of Gage and Hamilton, after a normalization, the curves tend to a circle.

As a consequence, the Lazutkin region, i.e. the region that contains convex caustics, gradually increases.

We describe some dynamical bifurcations in this process. In particular, we describe what happens with the family of period-two orbits and the "zig-zag" orbits.

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# FRAGILE AND STABLE CYCLES

LORENZO J. DÍAZ

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**


## Abstract

In a join paper with Bonatti we proved that every heterodimensional cycle yields robust cycles after arbitrarily small $C^1$-perturbations (i.e., existence of two hyperbolic sets of different indices whose invariant manifolds cyclically -and robustly- intersect). The previous result does not provide any information about these hyperbolic sets.

In this talk we will exhibit heterodimensional cycles such that the hyperbolic sets involved in the cycle cannot contain the initial saddles (in this case, we say that the cycle cannot be stabilized).

We also discuss the question about the stabilization of cycles.


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# DISSIPATIVE POLYGONAL BILLIARDS

PEDRO DUARTE
(JOINT WORK WITH JOÃO L. DIAS, JOSÉ P. GAIVÃO, GIANLUIGI DEL MAGNO AND DIOGO PINHEIRO)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

We study topological and ergodic properties of dissipative convex polygonal billiards, a class of systems introduced and named as 'Pinball billiads' by R. Markarian, E. J. Pujals, and M. Sambarino in [1]. Dissipative means the billiard map is defined based on a reflexion law where the angle of reflexion is strictly smaller than the incidence angle. Special atention is dedicated to the class of odd sided regular polygons.

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# SOME SYMMETRIC ORBITS FOR THE $n$-BODY PROBLEMS

DAVIDE FERRARIO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

We will describe extensions of finite rotation groups that are symmetry groups of some symmetric orbits for the $n$-body problem, trying to study some dynamical and topological features. The space of symmetric configurations turns out to be always homeomorphic to the complement of an arrangement of linear subspaces in $\mathbb{R}^n$, where suitable McGehee coordinates can be defined in order to study local dynamics around the total collision, and variational methods can be used to prove the existence of periodic orbits.

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# POSTER

## FIRST BIRKHOFF COEFFICIENT OF ELLIPTIC PERIODIC ORBITS FOR BILLIARD ON OVALS

GERALDO CESAR GONÇALVES FERREIRA

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

In this poster we calculate the First Birkhoff Coefficient of elliptic periodic orbits for billiard on ovals invariants by rotation. The orbits considered also are invariant by the same rotation. The value found is used to show stability of this orbits.

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# EQUILIBRIUM STATES FOR ROBUSTLY TRANSITIVE SYSTEMS

TODD FISHER

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We will prove existence and uniqueness of equilibrium states for a large class of potential functions associated to certain robustly transitive diffeomorphisms.

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# ACCUMULATION CASCADES IN FAMILIES OF STABLE SELF-INDUCED OSCILLATIONS IN LASERS: REGULARITIES IN PEAK-ADDING SEQUENCES GENERATED BY DDES, ODES, AND MAPS

JASON A. C. GALLAS

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Although intensively studied and applied since first introduced by Leibniz in 1676, differential equations still harbor a plethora of unanticipated and useful regularities. Such regularities are not properties of isolated solutions but become evident when contemplating the systematic way that very large *families of solutions,* periodic or not, organize themselves collectively in parameter space (in phase diagrams).

Phase diagrams reflect the self-organization of *stable* oscillations that are not difficult to observe experimentally for a variety of popular oscillators across all disciplines of natural sciences [1].

As examples, we discuss novel regularities, accumulations, found in the control space of an autonomous Duffing-like oscillator cleverly designed very recently as a proxy capable of bypassing noisy spectra polluting driven (non-autonomous) oscillators.

The Duffing-like proxy displays infinite peak-adding cascades of spirals like *periodicity hubs* [2] but of a remarkable new kind, consisting of an alternation of *disconnected pieces,* with ends containing the pair of stability structures, cusp and "fish"(see Fig. 1), known to be normal forms of cubic dynamics [3].

In addition, we also describe novel peak-adding cascades observed in dissipative systems described by delay-differential equations [4] and maps, with emphasis in semiconductor lasers [5].

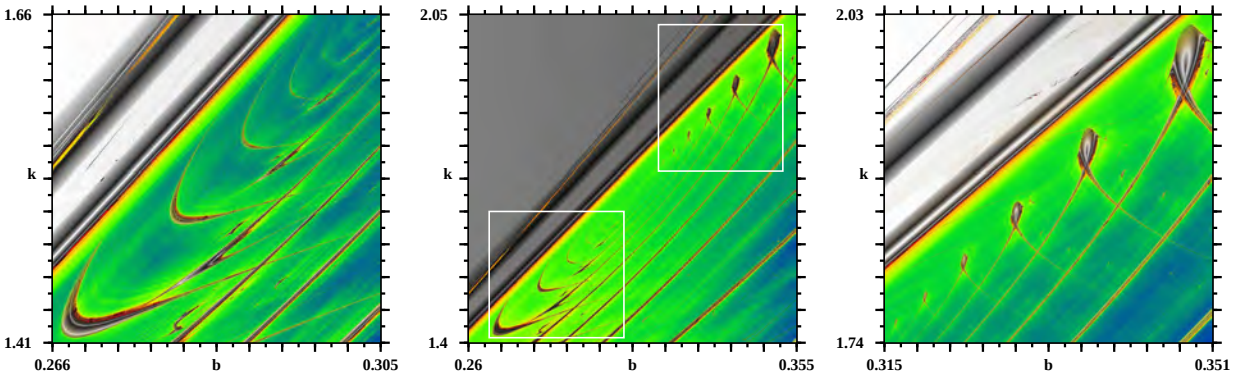


FIGURE 1. Phase diagrams showing that chaos thresholds (the edges of chaos separating colors from darker shadings) may contain an infinite spiral of non-continuous periodic phases [5].

**Support:**

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# SPECTRAL DECOMPOSITION IN NON-HYPERBOLIC DYNAMICS

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**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**


## Abstract

We aim for an understanding of the thermodynamic formalism in the context of smooth non-hyperbolic dynamics. We follow a classical approach to analyze its basic pieces and study homoclinic classes together with their thermodynamic properties. Focusing on a simple, by representative, example, we construct a local diffeomorphism that is a step skew product modeled over a horsehoe map that is naturally asociated to a heterodimensional cylce. This cycle gives rise to a homoclinic class on which the diffeomorphism is topologically transitive and partially hyperbolic. It can be conveniently studied in terms of an iterated function system of interval maps that are genuinely non-contracting. Our examples have topologically a rich fibre structure. Moreover, they exhibit a rich phase transition in the pressure function (coexistence of equilibrium states with positive entropies) that is associated to the Lyapunov exponents in the central direction. This phase transition is a consequence of a gap in the spectrum of central exponents.

Joint with L. Diaz (PUC Rio de Janeiro) and M. Rams (IM PAN Warsaw).


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# POSTER

## ACCESSIBILITY, SOME HYPERBOLICITY AND ROBUST TRANSITIVITY

ANA TÉRCIA MONTEIRO OLIVEIRA AND ALIEN HERRERA TORRES

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Under the condition of the property Some Hyperbolicity introduced by Pujals, Sambarino in [1] we prove two results. First that transitive, volume-preserving, accessible partially hyperbolic diffeomorphisms are robustly transitive in the space of volume-preserving diffeomorphisms. Second that accessible, topologically transitive partially hyperbolic diffeomorphisms are topologically mixing.

### References

[1] Pujals, E.; Sambarino, M.: *A sufficient condition for robustly minimal foliations.* Ergod. Th. Dynam. Sys. **26**, (2006) pp. 281-289

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# POSTER

## DENSE ORBITS OF VOLUME NON PRESERVING DYNAMICS

WALTER HUARACA VARGAS

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Let $N$ be a closed and orientable $n+1$-manifold and let $\phi$ be a smooth $\mathbb{R}^n$-action on $N$. A theorem due to Poincaré states that if $\phi$ is volume preserving, then almost every point of $N$ is recurrent under $\phi$. What can we say if $\phi$ does not preserve volume?

In this communication we examine the problem of existence of a dense orbit of $\phi$ on the manifold $N$. We also examine when the existence of this orbit implies that all the orbits of $\phi$ are dense, except a finite number. In particular we prove the following results:

If $\phi$ has not singular orbits and if $n = 2$, then every, except a finite number (one) of compact orbits of $\phi$, is dense.

If $\phi$ has a finite number of compact singular orbits of dimension one and if $n = 2$, then every orbit of $\phi$ is dense, except a finite number of orbits.

The above results give affirmative answers to Morse's conjecture in the cases that we study.

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# ARNOLD DIFFUSION VIA INVARIANT CYLINDERS AND MATHER VARIATIONAL METHOD

VADIM KALOSHIN

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**


## Abstract

The famous ergodic hypothesis claims that a typical Hamiltonian dynamics on a typical energy surface is ergodic. However, KAM theory disproves this. It establishes a persistent set of positive measure of invariant KAM tori. The (weaker) quasi-ergodic hypothesis, proposed by Ehrenfest and Birkhoff, says that a typical Hamiltonian dynamics on a typical energy surface has a dense orbit. This question is wide open.

In early 60th Arnold constructed an example of instabilities for a nearly integrable Hamiltonian of dimension $n > 2$ and conjectured that this is a generic phenomenon, nowadays, called Arnold diffusion. In the last two decades a variety of powerful techniques to attack this problem were developed. In particular, Mather discovered a large class of invariant sets and a delicate variational technique to shadow them. In a series of preprints: one joint with P. Bernard, K. Zhang and two with K. Zhang we prove Arnold's conjecture in dimension $n = 3$.


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# ON THE PLAQUE EXPANSIVITY CONJECTURE

SERGEY KRYZHEVICH

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

A partially hyperbolic dynamically coherent diffeomorphism of a smooth compact manifold is considered. We prove that if two central pseudo-trajectories are sufficiently precise and pointwise close, then one of these pseudo-trajectories belong to the center manifold of another one.

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# REGULARITY OF THE ENTROPY OF RANDOM WALKS ON HYPERBOLIC GROUPS

FRANÇOIS LEDRAPPIER

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

We show that the entropy of a random walk with finite support on a hyperbolic group depends in a Lipschitz way on the directing probability if its support remains constant.

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# GLOBAL OBSERVABLES AND THE QUESTION OF MIXING IN INFINITE ERGODIC THEORY

MARCO LENCI

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

Finding a satisfactory definition of mixing for dynamical systems preserving an infinite measure (in short, *infinite mixing*) is an important open problem. Virtually all the definitions that have been attempted so far use 'local observables', that is, functions that essentially only "see" finite portions of the phase space. We introduce the concept of 'global observable', a function that gauges a certain quantity throughout the phase space. This concept is based on the notion of infinite-volume average, which plays the role of the expected value of a global observable. Endowed with these notions, which are to be specified on a case-by-case basis, we give a number of definitions of infinite mixing. These fall in two categories: global-global mixing, which expresses the "decorrelation" of two global observables, and global-local mixing, where a global and a local observable are considered instead. Time permitting, we will see how these definitions respond on some examples of infinite-measure-preserving dynamical systems.

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# POSTER

## STICKINESS INFLUENCE IN A CONSERVATIVE BOUNCER MODEL: A SLOWING DOWN MECHANISM FOR FERMI ACCELERATION

ANDRÉ L. P. LIVORATI[1], CARL P. DETTMANN[2], IBERÊ L. CALDAS[1], AND EDSON. D. LEONEL[3]

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

The dynamics of a conservative bouncer model is studied. The approaches for the complete version and the simplified one are taken into account, and also a comparative with the standard mapping is made concerning chaotic properties related to a phase transition from integrable to non-integrable system for $\epsilon > \epsilon_c$. As we ranged this control parameter we would able to see the bifurcation of the fixed points as well stickiness around the stability islands. When this parameter is small enough, there is a large number of islands in the phase space, so a strong stickiness is observed and the Fermi acceleration phenomenon takes a really long time to be observed. As the control parameter is getting bigger the number os islands is getting smaller and this strong stickiness is no longer observed, leading the dynamics to experience Fermi acceleration in a faster way. We characterize this transition from strong to weak stickiness via finite time Lyapunov exponents and also scape rates in the velocity of an ensemble of particles. These results give support that the strong regime of sticky orbits is a mechanism that lead the dynamics to slow down their growth of the velocity, which characterizes Fermi acceleration.

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# ON DYNAMICS OF A FAMILITY OF STADIUM BILLIARDS: PHASE TRANSITIONS, FERMI ACCELERATION AND DECAY OF ENERGY

ANDRÉ L. P. LIVORATI[1], ALEXANDER LOSKUTOV[2], IBERÊ L. CALDAS[1], AND EDSON. D. LEONEL[3]

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

A point particle confined inside a stadium-like billiard is considered. In the static boundaries case, we construct a two-dimensional nonlinear area preserving mapping. Ranging the control parameters, we observed a transition from partial chaos to global, when the fixed points loose their stability. This transition is characterized by the defocusing mechanism. We also introduced a perturbation to the boundaries. Linearizing the unperturbed mapping, we found a critical value for the resonant velocity. For initial velocities smaller than the critical one, we observe a decreasing of the particles velocity. However, when initial velocity is larger than the resonant one, we observe a typical behavior of Fermi acceleration. When dissipation is introduced via inelastic collisions, we observe a phase transition from limited to unlimited energy growth in the regime of null dissipation.

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# ROBUST TRANSITIVITY FOR ENDOMORPHISMS

CRISTINA LIZANA ARANEDA AND ENRIQUE R. PUJALS

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We address the problem of the conditions under which an endomorphism having a dense orbit is such that a sufficiently close perturbed map also exhibits a dense orbit. For this purpose we give sufficient conditions, covering a large class of examples, for endomorphisms on the $n-$dimensional torus to be robustly transitive: the endomorphism must be volume expanding and any large connected arc must contain a point such that its future orbit belongs to an expanding region.

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# ENTROPY AND PRESSURE FOR ONE-DIMENSIONAL SPIN LATTICES WITH A GENERAL A PRIORI MEASURE: POSITIVE AND ZERO TEMPERATURE

ARTUR O. LOPES

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We generalize several results of the classical theory of Thermodynamic Formalism when the state space is a general compact metric space M. We consider the shift acting on $M^{\mathbb{N}}$ and a general a priori measure for defining the Ruelle operator. The choice of the a priori measure on M gives a restriction which is like the one we get from a shift of finite type over $\{1, 2, ..., d\}^{\mathbb{N}}$. We define entropy and by its very nature it is always a nonpositive number. If M is not finite there exists Gibbs states with arbitrary small negative value. We consider the Pressure problem and its relation with eigenfunctions and eigenprobabilities of the Ruelle operator. Among other things we analyze the case where temperature goes to zero and we show some selection results. Applications for the case when the state space is the set of integers are presented. Therefore, our general setting can be adapted (by compactification of the other setting) in order we can analyze the Thermodynamic Formalism for the Bernoulii space with countable infinite symbols. Moreover, the so called XY model also fits under our setting. In this last case M is the unitary circle $S^1$. We explore the differentiable structure of $(S^1)^{\mathbb{N}}$ considering potentials which are of class $C^2$ and the corresponding eigenfuntions.

This is a joint work with J. Mengue, J. Mohr and R. Souza

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# POSTER

## PROBLEMS ON DISCRETE TIME GENE TRANSCRIPTIONAL REGULATION NETWORKS

BEATRIZ CARELY LUNA OLIVERA

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

With advances in microbiology a lot of information about genes and their interactions is available. Analysis and tools are required to deal with this information. Usually, models consider differential equations or logical functions, however discrete time dynamical systems provide a good representation of gene regulatory networks, while biological questions on these networks open doors to interesting theoretical problems of dynamical systems in biology.

### 1. Gene transcriptional regulation networks

The central dogma of molecular biology proposes the correspondence between one gene and one protein. Gene expression is the process by which genes are transcribed into RNA, then, this information is used to synthesize proteins. Proteins are necessary in several functions performed by organisms. Often the protein produced by a certain gene is responsible of the activation or inhibition of another gene, that is to say, it promotes or restrict the expression respectively, and so on. These interactions can be represented into a network, where vertices symbolize genes and edges represent their interactions. It is possible to associate with every vertex in the network a level of produced protein, meaning that the network is dynamic. Regulatory networks are responsible of cell differentiation, metabolism, sensing signals, etc., understanding these networks could help to comprehend evolution, diseases, and even to control the cell in biotechnological processes.

### 2. Models of regulatory networks

Several formalisms have been used to model this type of systems, by instance: differential ordinary equations, piece-wise affine differential

equations, Petri nets, logical and Boolean networks [6]. Every model requires more or less information and recovers the dynamics with different levels of detail; nevertheless, dynamics must be equivalent given the biological suppositions [8].

We consider a network $N$, where vertices $V = \{1, \ldots, n\}$ represent genes, and edges $A \subseteq V \times V$ represent their interactions. We associate to each vertex $i$ on network a level of expression $x_i$.

In the simplest model, discrete time and levels of expression are considered [9], every gene take values $x_i^t \in \{0, 1\}$, and the next state $x_i^{t+1}$ of variable $i$, depends on the vertices interacting with $i$. The value of $x_i^{t+1}$ is obtained by means a logical function $x_i^{t+1} = f(x_1^t, \ldots, x_n^t)$. A state of the system is given by a combination of 0's and 1's of the $n$ variables, $\mathbf{x^t} = (x_1^t, \ldots, x_n^t)$, and the synchronous evolution of the system is given by the unique value $\mathbf{x^{t+1}} = (x_1^{t+1}, \ldots, x_n^{t+1})$.

On the other hand differential equations, piece-wise affine differential equations, and stochastic differential equations consider continuos time and phase space. If we consider two vertices $i$ and $j$, where $i$ regulates $j$, the number of molecules of protein $x_j$ produced per unit time is a function of the concentration of $i$ in its active form [2], $x_j = f(x_i)$. Usually, the input function $f(x_i)$ is a non linear function, in many cases this function is monotonous, increasing when $i$ is an activator and decreasing when $j$ is a repressor [6], which allows a good approximation by step functions.

## 3. Discrete-time regulatory networks

Now we consider discrete time but continuos phase space, this model was introduced in [10, 4, 7]. The evolution of the system is determined by the iteration of the piecewise affine transformation $F : [0, 1]^d \to [0, 1]^d$, where $d$ is the dimension of the network, and $F$ is given by

$$\mathbf{x}^{t+1} = F(\mathbf{x})_i = a_i \mathbf{x}_i^t + (1 - a_i) D(\mathbf{x}^t)_i, \tag{1}$$

with $i \in V$; $a_i \in [0, 1)$, is a constant, or contraction rate, which is related to speed of degradation of the activity of the unit $i$ in absence of interaction; $t$ takes positive integer values; and $D(\mathbf{x})_i$ is a piecewise constant function $D : [0, 1]^d \to [0, 1]^d$, whose definition relies on the architecture of the network, i.e. $D(\mathbf{x})_i$, depends only on the vertices influencing $i$.

The simplest case is the bi–modal activatory/inhibitory interaction, in these to each interaction $(i, j) \in A$ we associate a sign $s_{ij} = \{-1, 1, 0\}$, a threshold $T_{ij} \in (0, 1)$ and a weight of the interaction $K_{i,j} \geq 0$, $s_{ij}$ takes the value 0 whenever $K_{i,j} = 0$. Only two activation modes are possible: activation if $s_{ij} = 1$, and inhibition in the case $s_{ij} =$

$-1$. The interaction function $D : [0,1]^d \to [0,1]^d$, is then given by $D(\mathbf{x})_j = \sum_{i=1}^{d} K_{i,j}\, H(s_{i,j}(\mathbf{x}_i^t - T_{ij}))$, where $j$ stands for the vertex site, and $H : \mathbb{R} \to \{0,1\}$ is the Heavyside function defined as,

$$H(x) := \begin{cases} 1 & x > 0 \\ 0 & x \leq 0. \end{cases} \tag{2}$$

Multi–modal interactions can be also considered. As usual, *the orbit corresponding to the initial condition* $\mathbf{x}^0 \in [0,1]^d$, is the sequence $\{\mathbf{x}^t\}_{t\in\mathbb{N}} \subset [0,1]^d$ such that $\mathbf{x}^{t+1} := F(\mathbf{x}^t)$ for each $t \in \mathbb{N}$.

For this kind of systems, the local dynamics is regular, and the complexity of the dynamics comes from the interaction function, producing complex but still non–chaotic behavior.

## 4. Problems in regulatory networks

Some dynamical questions rise from regulatory networks, for instance: can we control the dynamics (fixed points, oscillatory behavior)?, are there some vertices dynamically more important than others?, what is the relation between topology and dynamics on network?, does exist dynamical modules to treat independently?, what are the observed behaviors under different parameters in the discrete-time model?, what is the relation between the different frameworks?. Proposed answers will be discussed.

**Acknowledgements:** I thank Cheryl Lynn Gad for her comments. This work was partially supported by grant PROMEP/103.5/11/4818.

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# SYNCHRONIZATION OF CHAOTIC MAPS IN DELAYED COMPLEX NETWORKS: ADVANCES, PERSPECTIVES AND APPLICATIONS

ARTURO C. MARTÍ

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

The synchronization of nonlinear units coupled on complex networks is one of the most hot topics in Nonlinear Physics. In this presentation we review recent advances, perspectives and applications on synchronization of chaotic maps in complex networks. We extend our previous works to Bernoulli, tent, sine, and bi-stable maps paying special attention to the relation between the topology and the distribution of the time delays.

The time delays associated with the links are either heterogeneous (Gaussian distributed) or homogeneous (i.e., the transmission speed is the same for all links; as a particular case we also study instantaneous interactions).

We show that, depending on the specific map, when the connectivity is low and the coupling strength is weak, the network displays an irregular oscillatory dynamics that is rather independent of the detailed network topology and delay distribution.

For large enough connectivity and coupling strength, the synchronization of the network is mainly determined by the delay times: when the delays are homogeneous the network exhibits collective synchronous oscillations; when the delays are heterogeneous, the network synchronizes in a steady-state.

We show that depending on the characteristics of the maps for some choices of initial conditions and time-delay distributions an island of synchronization appears, which corresponds with a periodic dynamical state of the system.

The addition of noise on the initial conditions is also studied. We compare numerical simulations with analytical results obtained for different maps.

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# POSTER

## PROPERTY SH AND ROBUST TRANSITIVITY

ANA TÉRCIA MONTEIRO OLIVEIRA

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

In the context of Partial Hyperbolicity, Pujals and Sambarino introduced in [1] an interesting property on the tangent bundle, named Property SH, which guarantees that the strong stable foliation is robustly minimal. We proved robust transitivity for a transitive diffeomorphism $f$ using only the condition of Property SH for it and its inverse. Moreover we give an scenario where these conditions are realized.

### References

[1] Pujals, E.; Sambarino, M.: *A sufficient condition for robustly minimal foliations.* Ergod. Th. Dynam. Sys. **26**, (2006) pp. 281-289

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# EXPANSIVITY OF ERGODIC MEASURES WITH POSITIVE ENTROPY

CARLOS MORALES (JOINT WITH ALEXANDER ARBIETO)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We prove that every ergodic invariant measure with positive entropy of a continuous map on a compact metric space is *expansive*, i.e., there is $\delta > 0$ such that the dynamical $\delta$-balls have measure zero. We use this property to prove, for instance, that the stable classes have measure zero with respect to any ergodic invariant measure with positive entropy. Moreover, continuous maps which either have countably many stable classes or are Lyapunov stable on their recurrent sets have zero topological entropy. We also apply our results to the Li-Yorke chaos.

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# A GENERALIZATION OF EXPANSIVITY.

CARLOS MORALES

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

We study dynamical systems for which at most $n$ orbits can accompany a given arbitrary orbit. For simplicity we call them $n$-expansive (or positively $n$-expansive if positive orbits are considered instead). We prove that these systems can satisfy properties of expansive systems or not. For instance, unlike positively expansive maps, positively $n$-expansive homeomorphisms may exist on certain infinite compact metric spaces. We also prove that a map (resp. bijective map) is positively $n$-expansive (resp. $n$-expansive) if and only if it is so outside finitely many points. Finally, we prove that a homeomorphism on a compact metric space is $n$-expansive if and only if it is so outside finitely many orbits. These last results extends previous ones for expansive systems.

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# INVARIANT DISTRIBUTIONS AND CONJUGACY FOR CIRCLE DIFFEOMORPHISMS.

ANDRÉS NAVAS

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

A remarkable theorem of A. Avila and A. Koksard establishes that minimal $C^\infty$ circle diffeomorphisms have no invariant distribution (of any regularity) other than the invariant measure. Using results of R. Douady and Y.-C. Yoccoz, we will show an analog result in low regularity (joint work with M. Triestino): no minimal circle diffeomorphism in the Denjoy class has 1-invariant distributions other than the invariant measure.

We relate this kind of results to the problem of determining the closure of the conjugacy class of a given (group of) diffeomorphism(s). I will show that the space of $\mathbb{Z}^d$-actions by $C^1$ circle diffeomorphisms is path connected. This is closely related to a prior result of N. Guelman and C. Bonatti that only applies to $\mathbb{Z}$ actions but gives information on local connectness.

Joint work with M. Ponce for the $C^2$ case will also be discussed if time permits.

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# POSTER

## NONDENSITY OF ORBITAL SHADOWING

ALEXEY OSIPOV

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

A diffeomorphism of a closed smooth manifold has some shadowing property if any sufficiently precise approximate trajectory of a certain type (a pseudo-trajectory) is in some sense close to some exact trajectory. Since the notion of "being close" can be formalized in several ways, various shadowing properties are considered. We will discuss standard shadowing (a pseudo-trajectory and an exact trajectory are pointwisely close), orbital shadowing (a pseudo-trajectory and an exact trajectory are close as sets), and weak shadowing (a pseudo-trajectory lies in some small neighborhood of an exact trajectory).

Bonatti, Diaz, and Turcat proved that standard shadowing is nondense with respect to the $C^1$-metric. Crovisier proved that weak shadowing is a $C^1$-generic property. We will prove nondensity of orbital shadowing with respect to the $C^1$-metric. Using techniques of Ilyashenko and Gorodetski, we construct a $C^1$-open domain such that any diffeomorphism from this domain has a partially hyperbolic invariant set, the dynamics on this set is conjugate with a so-called mild skew product, and hyperbolic periodic points with different indexes are dense in this set. We prove that this domain can be constructed in such a way that no diffeomorphism from this domain has orbital shadowing.

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# A TOY MODEL FOR FLOWS PRESENTING EQUILIBRIA ACCUMULATED BY REGULAR ORBITS

MARIA JOSE PACIFICO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We shall discuss some advances on flows with equilibria accumulated by regular orbits. We present a discrete model for these flows and prove mixing and exponential rate of mixing for such flows.

This corresponds to a joint work with R. Markarian and J. Vieitez.

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# A GENERAL APPROACH FOR CONSTRUCTING SRB-MEASURES FOR CHAOTIC ATTRACTORS

YAKOV PESIN

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

I discuss a general approach to the existence of SRB measures for local diffeomorphisms possessing chaotic attractors. I introduce a certain recurrence condition on the iterates of Lebesgue measure called effective hyperbolicity that is a version of LyapunovPerron regularity in the dissipative case. I will show that if the asymptotic rate of effective hyperbolicity is non-zero on a set of positive Lebesgue measure, then the system has an SRB measure. Some examples will be discussed.

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# ANOSOV TILINGS

ALBERTO A. PINTO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

In this talk we present the definition of a golden sequence $\{r_i\}_{i\in\mathbb{N}}$. These golden sequences have the property of being Fibonacci quasi-periodic and determine a tiling in the real line. We prove a one-to-one correspondence between:
(i) affine classes of golden tilings;
(ii) smooth conjugacy classes of Anosov diffeomorphisms, with an invariant measure absolutely continuous with respect to the Lebesgue measure, that are topologically conjugate to the Anosov automorphism

$$G_A(x, y) = (x + y, x)$$

(iii) solenoid functions.

**References and Literature for Further Reading**

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# DYNAMICS NEAR AN INVARIANT HORIZONTAL CIRCLE

SONIA PINTO-DE-CARVALHO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Billiards inside sufficiently differentiable ovals always have invariant rotational curves, but there are only three types of ovals such that the associated billiard map has an invariant horizontal circle in its phase-space: the circle, the constant width curves and some very special symmetric curves. In this joint work with Geraldo Gonçalves Ferreira (UFOP) and Sylvie Oliffson Kamphorst (UFMG), we study the dynamics of the billiard map of those three types of curves, near the horizontal circle and show that it is approached, from both sides, by other invariant rotational curves.

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# DIFFUSION OF MESSAGES AND MESSENGERS AND THE FORMATION OF MORPHOGEN GRADIENTS.

SILVINA PONCE DAWSON

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Diffusion is key to many physiologically relevant processes. It is particularly relevant for the formation of morphogen gradients which induce the differentiation of otherwise undifferentiated cells. One of the most widely studied morphogens is Bicoid (Bcd) which distribution is determinant for the organization of the anterior-posterior axis in *Drosophila* embryos [1]. About 80 minutes after egg deposition a stable Bcd gradient is established with larger Bcd concentrations at the anterior pole and an exponential decay towards the posterior end. This exponential distribution is consistent with the so called SDD model in which the protein is synthesized at the anterior end and subsequently diffuses and is degraded throughout the embryo. Within this model the Bcd diffusion coefficient is key to set the timescale over which the Bcd gradient forms and becomes stable. The Bcd diffusion coefficient was estimated using two optical techniques: Fluorescence Recovery After Photobleaching (FRAP) [2] and Fluorescence Correlation Spectroscopy (FCS) [3]. FRAP gave values that were too small to account for the establishment of the gradient within SDD model and the experimentally observed times. FCS gave several values one of which was compatible with the SDD model. The questions then arise of what is the correct estimate and whether the SDD model can still explain the gradient formation or not. In this talk I will show the results of a collaborative work with Lorena Sigaut, Alejandro Colman-Lerner and John E. Pearson in which we use a simple biophysical model to deal with this issue [4]. In particular, I will show that both the FRAP and the FCS estimates are correct and that their difference is perfectly understandable in terms of reactions of Bcd with binding sites [5]. It is the nonlinearities intrinsic to the reactions that are ultimately responsible for the disparity of the estimates one of which describes the diffusion

of individual Bcd molecules (the messengers) and the other one that of their population (the message) [6].

**Acknowledgements:** This research has been supported by UBA (UBACyT 20020100100064), ANPCyT (PICT 2010-1481 and PICT 2010-2767), CONICET (PIP 5131)

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# ON THE WORK OF JORGE LEWOWICZ ON EXPANSIVE SYSTEMS

RAFAEL POTRIE

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We will try to give an overview of one of the landmark results of Jorge Lewowicz: his classification of expansive homeomorphisms of surfaces. The goal will be to present the main ideas with the hope of giving evidence of the deep and beautiful contributions he made to dynamical systems. We will avoid being technical and try to concentrate on the tools constructed by Lewowicz to obtain these classification results such as Lyapunov functions and the concept of Persistence for dynamical systems. The main contribution that we will try to focus on is on his conceptual framework and approach to mathematics reflected on the previously mentioned tools and fundamentally in the delicate interaction between topology and dynamics of expansive homeomorphisms of surfaces he discovered in order to show his result.

*A person is judged by its mayor contribution*
Arab proverb paraphrased.

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# POSTER

## FLOWS WITH THE (ASYMPTOTIC) AVERAGE SHADOWING PROPERTY ON THREE-DIMENSIONAL CLOSED MANIFOLDS

RAQUEL RIBEIRO BARROSO PORTELA
(PHD IN MATHEMATICS. ADVISOR: MARÍA JOSÉ PACÍFICO)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We show that the interior of the set of flows with the (asymptotic) average shadowing property on a three-dimensional closed manifold is formed by transitive Anosov flows. This follows from an extension of a result in [2], where it is proved that any vector field robustly transitive is an Anosov flow. Our result generalizes the ones in [3] and [4], obtained for diffeomorphisms on surfaces.

### References

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# VOLUME GROWTH AND ENTROPY FOR PARTIALLY HYPERBOLIC DIFFEOMORPHISMS

RADU SAGHIN

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

I will present an inequality between metric entropy, Lyapunov exponents and volume growth in the unstable direction for $C^1$ partially hyperbolic diffeomorphisms. I will discuss different types of volume growth, connections with the de Rham cohomology and applications.

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# EXPONENTIAL CONVERGENCE OF THE SOLUTIONS OF THE TIME-PERIODIC HAMILTON-JACOBI EQUATION ON THE TORUS

HÉCTOR SÁNCHEZ MORGADO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We prove exponential convergence to time-periodic states of the solutions of time-periodic Hamilton-Jacobi equation on the torus, assuming that the Aubry set is the union of hyperbolic periodic orbits. The period of the limit solution is the least common multiple of the periods of the orbits of the Aubry set.

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# POSTER

## JACOBIAN CONJECTURE: A DIFFERENT APPROACH

JEAN VENATO SANTOS

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Let $F : \mathbb{R}^n \to \mathbb{R}^n$ be a polynomial local diffeomorphism and let $S_F$ denote the set of not proper points of $F$. The Jelonek's real Jacobian Conjecture states that if $\operatorname{codim}(S_F) \geq 2$, then $F$ is bijective. We prove a weak version of such conjecture establishing the sufficiency of a necessary condition for bijectivity. Furthermore, we generalize our result on bijectivity to semialgebraic local diffeomorphisms.

We will discuss how our result contributes for the Jacobian Conjecture, given a different point of view to attack this problem.

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# POSTER

## THRESHOLD CONDITION FOR VIRUS SPREAD IN A SIMPLE SIS PROPAGATION MODEL USING A DYNAMICAL SYSTEM APPROACH

ROBERTO BERNAL AND ALEXANDER SCHAUM

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Several approaches have been followed to study complex networks and their interactions. Among others, mean field theory [1], Markovian Analysis [3], and nonlinear dynamical system approaches [2] have been used to model and analyze the dynamics of viral propagation over a network. Given particular values of infection rate and virus death rate, we can pose several questions: Under which conditions a network infection can become global and how can it be controlled? How long does it take to disinfect a network?

In this work we show that it is possible to answer these questions for the case of a SIS-type propagation model. For this purpose we develop a nonlinear dynamical system that models viral propagation in an arbitrary network. Using fixed point stability theorems we show that it is possible to establish a general epidemic threshold condition for virus spread in a network, below which infections die out at an exponential rate. It is shown that this threshold corresponds to the inverse of the largest eigenvalue of the adjacency matrix associated to the network graph. Some possible control strategies are discussed, and numerical experiments are presented to illustrate the theoretic results.

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# THE UNICITY OF THE T-CONFORMAL MEASURE FOR HENON MAPS AT THE FIRST BIFURCATION

SAMUEL SENTI
(JOINT WORK WITH H. TAKAHASI)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We prove the existence and uniqueness of t-conformal measures for the Hénon map at the first bifurcation. In this talk we will stress the techniques needed to prove the uniqueness after giving a brief idea of how to obtain existence.

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# POSTER

## ON THE GEOMETRY OF HORSESHOES

CARLOS GUSTAVO TAMM DE ARAUJO MOREIRA AND WALISTON LUIZ LOPES RODRIGUES SILVA

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

The recurrent compact criterion was introduced by Moreira and Yoccoz to prove that "stable intersections of regular Cantor sets are dense in the region where the sum of their Hausdorff dimensions is bigger than 1."We adapt this concept to the context of horseshoes in dimension higher than 2 and prove that horseshoes with upper stable dimension bigger than 1 satisfy, typically and persistently, the adapted recurrent compact criterion. As consequences we present some geometric properties of these horseshoes. In particular, typically and persistently, horseshoes with upper stable dimension higher than 1 present blenders.

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# SINGULARITIES AND NONHYPERBOLIC MANIFOLDS DO NOT COINCIDE

NANDOR SIMANYI

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We consider the billiard flow of elastically colliding hard balls on the flat $\nu$-torus ($\nu \geq 2$), and prove that no singularity manifold can even locally coincide with a manifold describing future non-hyperbolicity of the trajectories.

As a corollary, we obtain the ergodicity (actually the Bernoulli mixing property) of all such systems, i.e. the verification of the Boltzmann-Sinai Ergodic Hypothesis.

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# SPACE FILLING CURVES, EXPANDING MAPS AND GEODESIC LAMINATIONS

VICTOR SIRVENT

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**


## Abstract

In this talk we consider a class of connected fractals that admit a space filling curve. We prove that these curves are Hölder continuous and measure preserving. To these space filling curves we associate geodesic laminations satisfying among other properties that points joined by geodesics have the same image in the fractal under the space filling curve. The laminations help us to understand the geometry of the curves. The construction of the laminations is associated to a family expanding dynamical system on the circle This family allow us to define expanding dynamical systems on the laminations. We explore the relations between the geometric properties of the laminations, the space-filling curves and the dynamical properties of the expanding maps.


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# RIGIDITY OF BIRKHOFF BILLIARDS

ALFONSO SORRENTINO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Despite being conceptually very simple, mathematical billiards presents a very rigid dynamics, which is completely determined by the geometry of the boundary and therein encoded. Trying to understand the extent of such rigidity and its implications to the dynamics is a formidable task, which lies behind many intriguing questions and conjectures.

In this talk I shall discuss the following problem: given two strictly convex billiards whose maps are conjugate near the boundary (i.e. for small angles), how are their shapes related?

In a joint work with Vadim Kaloshin we prove that if the conjugacy is sufficiently smooth, then the two domains must be similar, i.e. they are the same up to a rescaling and an isometry.

Our interest in this problem was arisen by a question posed by Guillemin and Melrose, which could be rephrased as follows: do the lengths of periodic orbits characterize the shape of the billiard domain? Or in a more colourful way: can one "hear" the shape of a billiard?

I shall describe how the above result can be used to provide an affirmative answer to this question.

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# SINGULARITIES IN GEOMETRICALLY DEFINED FOLIATIONS

JORGE SOTOMAYOR
JOINT WORK IN COLLABORATION WITH
R.A. GARCÍA AND D.L. DE SOUZA

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

Recent results concerning the patterns of principal curvature lines around partially umbilic curves on hypersurfaces in four dimensional euclidean spaces will be presented, extending results of R. García in [1], and extending to $\mathbb{R}^4$ results obtained in by C. Gutiérrez and J. Sotomayor for $\mathbb{R}^3$ in [2].

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# POSTER

## DOMINATED SPLITTINGS FOR COCYCLES WITH VALUES IN THE SEMIGROUP OF STOCHASTIC MATRICES

MANUEL STADLBAUER

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We prove that for a residual set of stochastic matrices over an ergodic automorphism, there always exists a dominated splitting. Furthermore, if the Lyapunov spectrum contains at least three points, then the Oseledets splitting is dominated and, in particular, the Lyapunov exponents vary continuously. This result might be seen as an extension of a result of Bochi and Viana to a class of non-accessible cocycles.

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# ON DETTMANN'S 'HORIZON' CONJECTURES

PÉTER NÁNDORI, DOMOKOS SZÁSZ, AND TAMÁS VARJÚ

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

Dettmann [1] formulated three conjectures for multidimensional dispersing billiards with infinite horizon. They, in particular, support the expectation that - similarly to 2D dispersing billiards with infinite horizon (cf. [2]) - superdiffusivity matrices can be simply calculated from geometric parameters of the model. First, the conjectures, talking about the tail probabilities of the free path lengths, are generalized to semi-dispersing billiards - possibly also with corner points, and then the first two of them are settled. The conclusions are applied to calculating the superdiffusivity covariance for the system of two hard balls on arbitrary dimensional tori and to certain cylindrical billiards.

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# TIRE TRACKS GEOMETRY, HATCHET PLANIMETER, MENZIN'S CONJECTURE, AND COMPLETE INTEGRABILITY

SERGEI TABACHNIKOV

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

This talk concerns a simple model of bicycle motion: a bicycle is a segment of fixed length that can move in the plane so that the velocity of the rear end is always aligned with the segment. The trajectory of the front wheel and the initial position of the bicycle uniquely determine its motion and its terminal position; the monodromy map sending the initial position to the terminal one arises. This circle mapping is a Moebius transformation, a remarkable fact that has various geometrical and dynamical consequences. Moebius transformations belong to one of the three types: elliptic, parabolic and hyperbolic. I shall outline a proof of a 100 years old conjecture: if the front wheel track is an oval with area at least Pi then the respective monodromy is hyperbolic. I shall also discuss the related Backlund-Darboux transformation, in the continuous and discrete settings, its complete integrability, and its surprising relation with the binormal (smoke ring, filament, local induction) equation, a much studied completely integrable PDE.

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# CENTRAL LYAPUNOV EXPONENTS OF PARTIALLY HYPERBOLIC DIFFEOMORPHISMS OF 3-TORUS

ALI TAHZIBI
(JOINT WORK WITH F. MICENA AND G. PONCE)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

In this talk we define uniform bounded density of foliations and prove some rigidity results for Lyapunov exponents of partially hyperbolic diffeomorphisms on 3-Torus. On the other hand, we construct example of partially hyperbolic diffeomorphisms on 3-torus with zero central Lyapunov exponent and non-compact central leaves.

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# SHADOWING LEMMA FOR PARTIALLY HYPERBOLIC SYSTEMS

SERGEY TIKHOMIROV

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

We say that diffeomorphism $f$ of a manifold $M$ is partially hyperbolic if tangent bundle of $M$ admits an invariant splitting $E^s + E^c + E^u$, such that $E^s$ and $E^u$ are uniformly hyperbolic and $E^c$ is not. If $E^c$ is empty diffeomorphism is uniformly hyperbolic.

Shadowing lemma says that in hyperbolic systems any pseudotrajectory can be shadowed by an exact trajectory. We introduce notion of central pseudotrajectory and prove that in partially hyperbolic systems any pseudotrajectory can be shadowed by a central pseudotrajectory.

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# A SURVEY ON NEW RESULTS ABOUT STATISTICAL PROPERTIES OF DETERMINISTIC AND RANDOM DYNAMICAL SYSTEMS

SANDRO VAIENTI

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

I will give a survey on some recents results on statistical properties of dynamical systems which I got with a few collaborators, and which could be of some interest in physical applications. In particular

- Optimal decay of correlations (lower bounds) for non-uniformly hyperbolic and expanding systems (joint with H Hu and H Zhang)
- Central limit theorem for the shrinking target problem (joint with N Haydn, M Nicol, L Zhang)
- Extreme value theory and Random Recurrence (joint with H Aytac and J Freitas)
- Escape rate and Metastability for Random Dynamical Systems (joint with W Bahsoun)

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# POSTER

## ABSOLUTE CONTINUITY FOR DIFFEOMORPHISMS WITH NON-COMPACT CENTER LEAVES

RÉGIS VARÃO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We study the measure-theoretical properties of center foliations of volume preserving partially hyperbolic diffeomorphisms with one-dimensional center direction.

Recent work of Avila, Viana, Wilkinson dealt with situations where the center leaves are compact or can be compactified in a suitable way.

Using different techniques we focus on the non-compact case and obtain very different conclusions.

For instance, in our context the disintegration of volume may be neither atomic nor Lebesgue. Such examples are found even among Anosov diffeomorphisms.

Moreover, even an Anosov may have absolutely continuous center foliation without being $C^1$-conjugate to its linearization.

We also discuss some rigidity results.

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# REMOVING ZERO CENTRAL LYAPUNOV EXPONENTS FOR PARTIALLY HYPERBOLIC DIFFEOMORPHISMS ON NILMANIFOLD

CARLOS H. VASQUEZ

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

We consider a diffeomorphism defined as the quotient of a Heissenberg automorphism by the natural lattice.

Such diffeomorphism is partially hyperbolic and its central Lyapunov exponent is zero.

In this talk we will discuss about the possibility to remove the zero central Lyapunov exponent using machinery "a la Shub-Wilkinson": derivating the unstable Lyapunov exponent and proving that it is a local maximum.

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# 1-DIM TILINGS AND BI-PARTITIONS

MACIEJ P. WOJTKOWSKI

**Montevideo Dynamical Systems Conference 2012**
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**Abstract**

Bi-partitions are partitions of the 2-dim torus by two parallelograms. They give rise to 2-periodic tilings of the plane, and further to 1-dim tilings which have a host of well known combinatorial properties, e.g. these are Sturmian sequences.

When a bi-partition is a Markov partition for a hyperbolic toral automorphism (= Berg partition), the tilings are substitution tilings. The substitutions preserving Sturmian sequences are known to have the "3-palindrome property". The number of different substitutions was determined by Seebold '98, and the number of nonequivalent Berg partitions by Siemaszko and Wojtkowski '11. The two formulas coincide.

Using tilings we give a simpler proof for the last result. It shows that every combinatorial substitution preserving a Sturmian sequence is realized geometrically as a Berg partition.

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# APPENDIX

## LATE COMPILED ABSTRACTS

# ERGODICITY OF THE WEIL-PETERSSON GEODESIC FLOW

KEITH BURNS

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

The lecture will describe the Weil Petersson metric on the moduli space of a surface and (at least some of) the ideas that go into the proof that its geodesic flow is ergodic.

This is joint work with Howard Masur and Amie Wilkinson.

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# ANGULAR MOMENTUM AND HORN'S PROBLEM

ALAIN CHENCINER

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**


## Abstract

The central configurations of n point masses are those configurations which admit periodic rigid motions when submitted to the Newtonian attraction. For example, while completing in 1772 the first full reduction of the symmetries in the 3-body problem, Lagrange proved that the only non-colinear central configuration of 3 positive masses is the equilateral triangle. Such rigid motions necessarily take place in an euclidean space E of even dimension 2p and, the initial configuration being given, they are uniquely defined by the choice of a complex structure on this space (Albouy-Chenciner, 1998). Introducing two moment maps related to Horn problems, one in dimension p and one in dimension 2p, and using a deep combinatorial lemma of Fulton-Fomin-Li-Poon, we answer the following question: fixing an initial central configuration of n bodies in E, what can be said of the mapping which, to each complex structure, associates the spectrum of the angular momentum (transformed into an antisymmetric endomorphism via the euclidean structure) of the associated relative equilibrium? Part of this work was done with Hugo Jimenez Perez.


## References


[1] A. Chenciner *The angular momentum of a relative equilibrium*, arXiv:1102.0025, final version to appear in D.C.D.S.

[2] A. Chenciner & Hugo Jiménez-Pérez *Angular momentum and Horn's problem*, arXiv:1110.5030, submitted to the Moscow mathematical Journal.

[3] S. Fomin, W. Fulton, C.K. Li, Y.T. Poon, *Eigenvalues, singular values, and Littlewood-Richardson coefficients*, Amer. J. Math. 127, no. 1, 101–127 (2005)



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# ON GEOMETRICAL PROPERTIES OF HORSESHOES IN ARBITRARY DIMENSIONS

CARLOS "GUGÚ" MOREIRA (JOINT WORK WITH WALISTON SILVA)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

## Abstract

We prove that, for typical horseshoes in ambient dimension larger than 2 for which the Hausdorff dimension of the intersection between the horseshoe and some stable leaf is larger than one, the image by typical $C^1$ real functions of the intersection between the horseshoe and every stable leaf persistently contains intervals. We also prove that these horseshoes are typically blenders (in the sense of Bonatti and Diaz). We will also discuss how to adapt the techniques developed in this work in order to show that, for typical horseshoes in dimension 3, the Hausdorff dimensions of its stable and unstable Cantor sets, and of the whole horseshoe are continuous with respect to the dynamics.

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# EXPANSIVE AND TOPOLOGICALLY STABLE GEODESIC FLOWS: FROM DYNAMICS TO GLOBAL GEOMETRY AND RIGIDITY

RAFAEL OSWALDO RUGGIERO

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**

**Abstract**

The works of Jorge Lewowicz about expansive homeomorphisms of compact surfaces had remarkable impact in the theory of geodesic flows without conjugate points. We shall make a survey of results about expansive and weakly stable geodesic flows in compact manifolds without conjugate points, starting from Lewowicz's results and views about expansive dynamics in low dimensions.

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# POSTER

## INFINITESIMAL LYAPUNOV FUNCTIONS FOR SINGULAR FLOWS

LUCIANA SALGADO (JOINT WORK WITH VITOR ARAUJO)

**Montevideo Dynamical Systems Conference 2012**
**From 13th. to 17th. August**


### Abstract

In this work, joint with Vitor Araujo, we present an extension of the notion of infinitesimal Lyapunov function to singular flows, and from this technique we deduce a characterization of partial hyperbolic sets. When the maximal invariant set of a neighborhood is contained into non-wandering set of the flow, we obtain a similar result for sectional hyperbolic sets. In abscence of singularities, we can also characterize uniform hyperbolicity.

These conditions might be expressed using the vector field X and its space derivative $DX$ together with an infinitesimal Lyapunov function only and are reduced to checking that a certain symmetric operator is positive definite on the trapping region. These results extend the Lewowiczs the- orem to the set of singular flows, by following Wojtkowski and Potapov.


### References

[1] V. Araujo, L. S. Salgado. *Infinitesimal Lyapunov functions for singular flows.* Preprint arXiv:1201.2550v2 (2012)

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